MATH 441
QUIZ 2
FALL 2013

NAME Key

SHOW ALL WORK

① Find the adjoint of $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 3 \\ 2 & -2 & 0 \end{bmatrix}$

$$\text{MATRIX OF COFACTORS} = \begin{bmatrix} \begin{vmatrix} 3 & 3 \\ -2 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} \\ -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} & \begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} \\ \begin{vmatrix} 1 & -1 \\ 3 & 3 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 6 & 6 & -8 \\ 2 & 2 & 8 \\ 6 & -10 & 8 \end{bmatrix}$$

$$\text{Adj}(A) = \begin{bmatrix} 6 & 2 & 6 \\ 6 & 2 & -10 \\ -8 & 8 & 8 \end{bmatrix}$$

② Give at least one reason why the set of all $2 \times 2$ singular matrices with standard operations is not a vector space

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

NOT CLOSED UNDER ADDITION

③ Rather than use the standard definitions of addition and scalar multiplication in $\mathbb{R}^2$, suppose

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$c(x, y) = (c^2 x, c^2 y).$$

Show that with these definitions $\mathbb{R}^2$ is not a vector space

$$(c+d)(x, y) = ((c+d)^2 x, (c+d)^2 y)$$
$$= ((c^2 + 2cd + d^2)x, (c^2 + 2cd + d^2)y)$$

But

$$c(x,y) + d(x,y) = (c^2 x, c^2 y) + (d^2 x, d^2 y)$$
$$= ((c^2 + d^2)x, (c^2 + d^2)y)$$

AND THESE ARE NOT EQUAL
(ALTHOUGH THE SHOULD BE.)

④ Write $(2,5,7)$ as a linear combination of $\vec{v}_1 = (1,2,3)$, $\vec{v}_2 = (0,1,2)$ and $\vec{v}_3 = (-2,0,1)$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 2 \\ 2 & 1 & 0 & 5 \\ 3 & 2 & 1 & 7 \end{array}\right] \xrightarrow[-3R_1+R_3]{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 2 \\ 0 & 1 & 4 & 1 \\ 0 & 2 & 7 & 1 \end{array}\right] \xrightarrow{-2R_2+R_3} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 2 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

$$\xrightarrow{-R_3} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 2 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow[2R_3+R_1]{-4R_3+R_2} \left[\begin{array}{cccc} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$(2,5,7) = 4(1,2,3) - 3(0,1,2) + 1\cdot(-2,0,1)$$